Math 141 **Test #1 Solutions** **Fall 2004**

1. (a) Method 1: The inequality says that x is less than 2 units from 5. Therefore, $3 < x < 7$.

   (b) Method 2: Rewrite it as the inequalities

$$-2 < x - 5 < 2$$

   and add 5 to all three members to get $3 < x < 7$.

2. Complete the square in x to get $x^2 + 2x + 1$ and complete the square in y to get $y^2 - 4y + 4$. So the equation becomes

$$(x^2 + 2x + 1) + (y^2 - 4y + 4) - 4 = 1 + 4,$$

$$(x + 1)^2 + (y - 2)^2 = 9 = 3^2.$$

   So the center of the circle is $(-1, 2)$ and the radius is 3.

3. The slope of the given line is $3/4$, so the slope of the perpendicular line must be $-4/3$. Then use the point-slope form:

$$y - 5 = -\frac{4}{3}(x - 2).$$

4. Because of the square root, we must have $x^2 \leq 100$, so the domain is $-10 \leq x \leq 10$. That is the closed interval $[-10, 10]$. As x ranges from -10 to 0, $f(x)$ ranges from 0 to 10, and as x ranges from 0 to 10, $f(x)$ ranges from 10 to 0. Therefore, the range is the closed interval $[0, 10]$. Also, note that the graph is the upper semicircle centered at $(0, 0)$ with radius 10.

5. Use a calculator to evaluate the expression at the given values of x:

x	0.9	0.99	0.999	1.001	1.01	1.1
$f(x)$	0.02633	0.002513	0.0002501	-0.0002499	-0.002488	-0.02382

   Apparently, the limit is 0.

6. (a) Since there is no division by 0, just substitute $x = 3$ into the expression and get the limit of 5.

   (b) Factor the numerator as $(x - 3)^2$ and then cancel a common factor of $x - 3$ with the denominator. The remaining expression is $x - 3$, whose limit is 0 as $x \to 0$.

   (c) Multiply the numerator and denominator by the conjugate $\sqrt{x} + 1$ to get the expression

$$\frac{x - 1}{(x - 1)(\sqrt{x} + 1)}.$$

   Then cancel the common factor of $x - 1$, leaving $\frac{1}{\sqrt{x}+1}$, whose limit is $1/2$ as $x \to 1$.

(d) This limit does not exist because x is approaching 0 through negative values, but you cannot evaluate $\sqrt{x}$ at negative values of x.

7. (a) False. At $x = 2$, one formula yields the value 5 and the other yields the value 6.

   (b) True. Only the expression $2x+1$ applies to this interval and it is continuous.

   (c) False, because of the discontinuity at $x = 2$.

   (d) True. All that matters on the closed interval $[0, 2]$ is that f be continuous on the open interval $(0, 2)$ and that the right limit $\lim_{x\to 0^+} f(x)$ exist at 0 and the left limit $\lim_{x\to 0^-} f(x)$ exist at 2, which they do.

8. The denominator goes to 0 while the numerator does not. Therefore, the limit, if it exists, is not finite. But is it $+\infty$ or $-\infty$? Compute the left and right limits at 1:

$$\lim_{x\to 1^-} \frac{x^2}{x-1} = -\infty$$

because $x^2 > 0$ and $x - 1 < 0$ when $x < 0$, and

$$\lim_{x\to 1^+} \frac{x^2}{x-1} = +\infty$$

because $x^2 > 0$ and $x - 1 > 0$ when $x > 0$. Therefore, the limit does not exist.